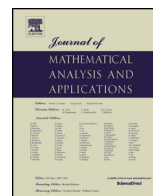




Contents lists available at ScienceDirect

Journal of Mathematical Analysis and Applications

www.elsevier.com/locate/jmaa



Sharp Hardy constants for annuli

F.G. Avkhadiev

Lobachevsky Inst. of Math. and Mech., Kazan Federal University, Kremlevskaya str., 35, 420008, Kazan, Russia

ARTICLE INFO

Article history:

Received 2 April 2017

Submitted by E. Saksman

Keywords:

Hardy inequality

Conformal modulus

Hypergeometric function

Rellich type inequality

ABSTRACT

We consider the Hardy inequality in canonical doubly connected plane domains. For any annulus A we determine sharp Hardy's constant $c_2(A)$ in function of conformal modulus $M(A)$. Namely, for any annulus A with fixed conformal modulus $M(A) = M$ we prove that

$$c_2(A) = \begin{cases} 1/4, & \text{if } M \in (0, M^*]; \\ \gamma(2 - \gamma)/4, & \text{if } M \in (M^*, \infty), \end{cases}$$

where $\gamma = \gamma(M) \in (1, 2)$. The critical modulus $M^* \approx 0.57298$ and the values of $\gamma(M)$ are found as roots of certain equations, containing the Gauss hypergeometric functions. In particular, we show that the sharp Hardy constants $c_2(A)$ depend on M continuously and that they tend to zero as $M \rightarrow \infty$. In addition, we describe an application of results to a Rellich type inequality.

© 2018 Elsevier Inc. All rights reserved.

1. Introduction

Let $\Omega \subset \mathbb{C}$ be a plane domain such that $\Omega \neq \mathbb{C}$. We consider functions $\varphi : \Omega \rightarrow \mathbb{R}$ and the following Hardy inequality

$$\iint_{\Omega} |\nabla \varphi(z)|^2 dx dy \geq c_2(\Omega) \iint_{\Omega} \frac{\varphi^2(z)}{(\text{dist}(z, \partial\Omega))^2} dx dy, \quad \forall \varphi \in C_0^\infty(\Omega), \quad (1)$$

where $z = x + iy$, $\text{dist}(z, \partial\Omega) := \inf_{\zeta \in \partial\Omega} |z - \zeta|$ is the distance from a point $z \in \Omega$ to the boundary of the domain. We suppose that the quantity $c_2(\Omega)$ is defined as the best possible constant, i.e.

$$c_2(\Omega) = \inf_{\varphi \in C_0^\infty(\Omega), \varphi \neq 0} \frac{\iint_{\Omega} |\nabla \varphi(z)|^2 dx dy}{\iint_{\Omega} \varphi^2(z) (\text{dist}(z, \partial\Omega))^{-2} dx dy}. \quad (2)$$

E-mail address: avkhadiev47@mail.ru.